

# Cosmological constraints from CMB distortion

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We examine bounds on adiabatic and isocurvature density fluctuations from  $\mu$ -type spectral distortions of the cosmic microwave background (CMB). Studies of such distortion are complementary to CMB measurements of the spectral index and its running, and will help to constrain these parameters on significantly smaller scales. We show that a detection on the order of  $\mu \sim 10^{-7}$  would strongly be at odds with the standard cosmological model of a nearly scale-invariant spectrum of adiabatic perturbations. Further, we find that given the current CMB constraints on the isocurvature mode amplitude, a nearly scale-invariant isocurvature mode (common in many curvaton models) cannot produce significant  $\mu$ -distortion. Finally, we show that future experiments will strongly constrain the amplitude of the isocurvature modes with a highly blue spectrum as predicted by certain axion models.

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## I. INTRODUCTION

The scale-invariant formulation of the power spectrum of primordial fluctuations is highly successful [1]. The observational results of both cosmic microwave background (CMB) and large scale structure (LSS) studies are modeled superbly by a scale-invariant spectrum. However, observational data for the power spectrum on small scales (smaller than 0.1 Mpc) is lacking, prompting discussions of a possible deviation from scale invariance on small scales. For example, many inflationary models predict a running spectral index, which may develop a large deviation from a scale invariant spectrum at large CMB multipoles  $\ell$  (for a survey of theoretical motivations and observational prospects for a running index, see [2]).

Isocurvature fluctuation modes can also produce deviations from a scale invariant spectrum. For example, isocurvature fluctuations are generated in the axion dark matter model [3] and the curvaton scenario [4]. If the spectrum is blue, the isocurvature modes contribute additional power to the density fluctuations on small scales, although the isocurvature mode remains subdominant on large scales. Current observational constraints allow models with a mixture of scale-invariant adiabatic and isocurvature modes with a blue spectrum [5], with isocurvature modes constrained to contribute at most 10% on large scales (pure isocurvature models are already observationally ruled out [6]).

Currently, the running of the power spectrum index for adiabatic modes and the amplitude of isocurvature modes are constrained by CMB and large scale structure [1]. For the running power spectrum, the best fit value for WMAP and large scale structure is  $-0.022 \pm 0.020$ . Furthermore, the amplitude ratio of the isocurvature mode to the adiabatic mode at  $k = 0.002 \text{Mpc}^{-1}$  is 0.13 and

0.011 for the uncorrelated CDM isocurvature case and the correlated one, respectively. However these constraints can be vastly improved by measurements of the fluctuations on smaller scales.

The measurement of the CMB spectrum distortion from the perfect blackbody shape is a powerful tool for producing constraints on small scale density fluctuations before the recombination epoch. The energy of the density fluctuation on small scales is released into CMB photons via Silk damping [7]. At a very high redshift ( $z > 10^6$ ), the released energy is quickly thermalized by double-Compton and Compton scattering, and the spectrum of the CMB photons maintains a blackbody shape. However, as the universe evolves, these scatterings become less effective and cannot establish thermalization of the released energy. As a result, released energy is imprinted on the CMB spectrum as CMB distortions from the blackbody shape [8, 9]. Such distortions may be classified into two types,  $\mu$ -distortion and  $y$ -distortion [10]. While  $\mu$ -distortion is produced from the double Compton scattering decoupling ( $z \sim 10^6$ ) to the thermalization decoupling by Compton scattering ( $z \sim 10^5$ ),  $y$ -distortion is generated from thermalization decoupling by Compton scattering to the recombination epoch. In other words, by measuring these distortions, we can learn about the density fluctuations on scales smaller than the Silk damping scale at these redshifts, up to  $k \sim 10^5 \text{Mpc}^{-1}$ .

The current constraints on these distortions, obtained by COBE FIRAS, are  $|\mu| < 9 \times 10^{-5}$  and  $y < 1.5 \times 10^{-5}$  [11]. Proposals for future space missions such as PIXIE have the potential to provide dramatically tighter constraints on both types of distortion, with projected detection levels of  $|\mu| \sim 5 \times 10^{-8}$  or  $y \sim 10^{-8}$  by PIXIE at the  $5\sigma$  level [12]. It was recently pointed out that PIXIE can detect the CMB distortion due to the dissipation of the primordial fluctuations with  $n_s = 0.96$  at a  $1\sigma$  level, because the distortions for  $n_s = 0.96$  are  $\mu \sim 8 \times 10^{-9}$  [13–15].

In this paper, we investigate  $\mu$ -distortions caused by the two types of fluctuations described above. One is the

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primordial fluctuation with running spectral index and the other is an additional power-law fluctuation which is sub-dominant on large scales. One of the motivations for the latter type is the CDM or baryon isocurvature mode with a blue spectrum. An extremely blue spectrum ( $n_{iso} > 2$ ) can be generated in the axion model suggested in [16]. Although the density fluctuations of CDM and baryons due to the isocurvature mode can survive Silk damping, the photon density fluctuations produced by the isocurvature mode suffer Silk damping [17] and this dissipation can produce  $\mu$ -distortion.

## II. DISTORTIONS FROM CURVATURE PERTURBATIONS

The energy injection in the early universe produces CMB spectral distortions. The evolution of the spectral distortions depends on the energy injection rate and the time scale of the thermalization process. In particular, in the regime between  $z \sim 10^6$  and  $z \sim 10^5$ ,  $\mu$ -distortion is created by the balance between the energy injection and the double Compton scattering. The time evolution of  $\mu$ -distortion due to this energy injection is given by [9]

$$\frac{d\mu}{dt} = -\frac{\mu}{t_{DC}(z)} + 1.4 \frac{dQ/dt}{\rho_\gamma}, \quad (1)$$

where  $t_{DC}$  is the time scale for double Compton scattering

$$t_{DC} = 2.06 \times 10^{33} \left(1 - \frac{Y_p}{2}\right)^{-1} (\Omega_b h^2)^{-1} z^{-9/2} s, \quad (2)$$

where  $Y_p$  is the primordial helium mass fraction. The solution to this evolution is

$$\begin{aligned} \mu &= 1.4 \int_{t(z_1)}^{t(z_2)} dt \frac{dQ/dt}{\rho_\gamma} e^{-(z/z_{DC})^{5/2}} \\ &= 1.4 \int_{z_1}^{z_2} dz \frac{dQ/dz}{\rho_\gamma} e^{-(z/z_{DC})^{5/2}}, \end{aligned} \quad (3)$$

where

$$z_{DC} = 1.97 \times 10^6 \left(1 - \frac{1}{2} \left(\frac{Y_p}{.24}\right)\right)^{-2/5} \left(\frac{\Omega_b h^2}{.0224}\right)^{-2/5} \quad (4)$$

The dissipation of acoustic waves in the photon-baryon plasma due to Silk damping injects energy into the CMB and causes the CMB spectral distortions.

We can write the energy density perturbation of the acoustic waves as

$$Q = \frac{\rho_\gamma}{2} [c_s^2 \langle \delta_\gamma(\mathbf{x})^2 \rangle + \langle v(\mathbf{x})^2 \rangle]. \quad (5)$$

The density and velocity perturbation of photons are

$$\langle \delta_\gamma(\mathbf{x})^2 \rangle = \int \frac{d^3 k}{(2\pi)^3} P_\gamma(k) \quad (6)$$

$$\langle v_\gamma(\mathbf{x})^2 \rangle = \int \frac{d^3 k}{(2\pi)^3} P_v(k). \quad (7)$$

The power spectrum  $P_i(k)$ , where the subscript  $i$  represents  $\gamma$  and  $v$ , is related to the primordial power spectrum  $P_\gamma^i(k)$  by

$$P_i(k) = \Delta_i^2(k) P_\gamma^i, \quad (8)$$

where the transfer function  $\Delta_i(k)$  for modes well inside the horizon satisfies the relation

$$\Delta_\gamma(k) \approx 3 \cos(kr_s) e^{-k^2/k_D^2}, \quad (9)$$

and

$$\Delta_v(k) \approx 3c_s \sin(kr_s) e^{-k^2/k_D^2}, \quad (10)$$

with the sound horizon  $r_s$  given by

$$\begin{aligned} r_s(z) &= \frac{2}{3} \frac{1}{k_{eq}} \sqrt{\frac{6}{R(z_{eq})}} \\ &\times \ln \left( \frac{\sqrt{1+R(z)} + \sqrt{R(z_{eq})+R(z)}}{\sqrt{R(z_{eq})}+1} \right), \end{aligned} \quad (11)$$

and  $k_D$  is the diffusion scale

$$\frac{1}{k_D^2} = \int_z^\infty dz \frac{c(1+z)}{6H(1+R)n_e \sigma_T} \left( \frac{R^2}{1+R} + \frac{16}{15} \right). \quad (12)$$

In the above,  $R$  is the baryon energy density

$$R \equiv \frac{3\rho_b}{4\rho_\gamma}, \quad (13)$$

$n_e$  is the free electron number density (before the recombination epoch, it is given by  $n_e(z) = (n_{H0} + 2n_{He0})(1+z)^3 \equiv n_0(1+z)^3$ ), and  $\sigma_T$  is the Thomson scattering cross-section. During radiation domination the diffusion scale becomes

$$\begin{aligned} k_D &= A_D^{-1/2} (1+z)^{3/2}; \\ A_D &= \frac{8c}{135H_0\Omega_r^{1/2} n_{e0} \sigma_T} = 5.92 \times 10^{10} \text{Mpc}^2. \end{aligned} \quad (14)$$

Now, we can write the energy density in acoustic waves is

$$Q = \Delta_Q(k)^2 P_\gamma^i, \quad (15)$$

where

$$\Delta_Q(k) = \frac{3c_s}{\sqrt{2}} e^{-k^2/k_D^2}. \quad (16)$$

The primordial power spectrum can then be related to the power spectrum for the curvature in the co-moving gauge,  $\zeta$ , via [14]

$$P_\gamma^i = \frac{4}{(\frac{2}{5}R_\nu + \frac{3}{2})^2} P_\zeta \approx 1.45 P_\zeta \quad (17)$$

where  $R_\nu$  is the neutrino energy density

$$R_\nu = \frac{\rho_\nu}{\rho_\gamma + \rho_\nu} \approx .4. \quad (18)$$

The power spectrum of the curvature is parameterized as [1]

$$P_\zeta = \frac{A_\zeta 2\pi^2}{k^3} \left( \frac{k}{k_0} \right)^{n_s - 1 + \frac{1}{2} \ln(\frac{k}{k_0}) \frac{dn_s}{d\ln k}} \quad (19)$$

with normalized amplitude  $A_\zeta = 2.4 \times 10^{-9}$ , and pivot scale  $k_0 = .002 \text{Mpc}^{-1}$ , the spectral index is  $n_s$ , and the running of the spectral index is  $\alpha_s \equiv dn_s/d\ln k$ .

**The energy release per unit redshift is given by**

$$\frac{dQ/dz}{\rho_\gamma} = - \int \frac{d^3 k}{(2\pi)^3} P_\gamma^i(k) \frac{d\Delta_Q^2}{dz}. \quad (20)$$

We then solve Eq.(3) while scanning over the  $\alpha_s - n_s$  plane using the values  $Y_p = .24$ ,  $\Omega_b h^2 = .0224$ , along with the previously mentioned values for  $A_\zeta$ ,  $A_D$ , and  $t_{DC}$ . We have calculated  $\mu$  for the injection interval  $z_1 = 2 \times 10^6$  to  $z_2 = 5 \times 10^4$ , and the results are displayed in Fig.(1).

The current WMAP7 bounds are given in Table 7 of [1]. Roughly, for no tensor modes and no running ( $\alpha_s = 0$ ),  $n_s = .968 \pm .012$ ; including running they find  $n_s = 1.008 \pm .042$  with  $\alpha_s = -.022 \pm .02$ . Including tensor modes gives  $n_s = 1.07 \pm .06$ ,  $\alpha_s = -.042 \pm .024$ , and  $r < .49$  (95%CL). The running we have plotted in Fig.(1) is significantly smaller, showing the constraining ability of such a precise  $\mu$  measurement.

With regards to a measurement of the running,  $\alpha_s$ , Planck may be able to reach sensitivities on the order of  $|\alpha_s| \simeq .005$ , and we see that measurements of  $\mu$  at the level of  $10^{-8}$ , in tandem with complementary measurements of  $n_s$ , would be at least competitive with such a measurement.

As we have stated above, the measurement of  $\mu$  may be comparable to or possibly supersede bounds on the spectral index or its running given by CMB observations such as WMAP and Planck. However, it should be emphasized that the  $\mu$  measurement would give information about such cosmological parameters on much smaller scales, and from effects occurring at much higher redshifts. Thus, making such measurements is quite attractive, and would allow us to reach previously unexplored distance and time scales for cosmological observations.

### III. DISTORTIONS FROM ISOCURVATURE PERTURBATIONS

The existence of additional density fluctuations in the early universe is another possible heat source for the spectral distortion. Candidates of such additional density fluctuations are isocurvature perturbations.

We define the isocurvature perturbations between photons and CDM (or baryons) as

$$S \equiv \delta_i - \frac{3}{4} \delta_\gamma, \quad (21)$$

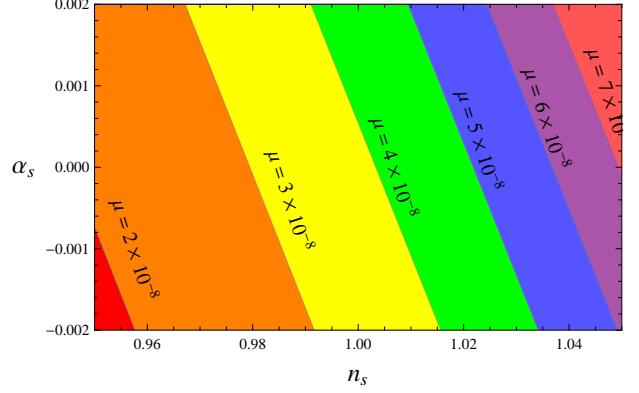


FIG. 1. Values of  $\mu$  generated by energy injection due to the damping of acoustic oscillations in the  $\alpha_s$ - $n_s$  plane for adiabatic modes with amplitude  $2.4 \times 10^{-9}$ .

where the subscript  $i$  is  $c$  for the CDM isocurvature and  $b$  for baryon isocurvature. We parameterized the power spectrum of  $S$  similarly to that of the curvature perturbations

$$P_{iso} = \frac{A_{iso} 2\pi^2}{k^3} \left( \frac{k}{k_0} \right)^{m_{iso}-1} \quad (22)$$

where we have neglected the possibility of a running isocurvature spectral index. The value of the amplitude,  $A_{iso}$ , is constrained by CMB data to be  $A_{iso} \lesssim .1 A_\zeta$ .

The power spectrum of the isocurvature perturbations for photon density fluctuations or velocity is given by

$$P_i(k) = \Delta_{i,iso}^2(k) P_{iso}(k), \quad (23)$$

where  $\Delta_{i,iso}$  which can be written, well inside the horizon, as [17],

$$\begin{aligned} \Delta_{\gamma,iso}(k) &\approx -\sqrt{6} \left( \frac{k_{eq}}{k} \right) \sin(kr_s) e^{-k^2/k_D^2} \\ \Delta_{v,iso}(k) &\approx -\sqrt{6} c_s \left( \frac{k_{eq}}{k} \right) \cos(kr_s) e^{-k^2/k_D^2} \end{aligned} \quad (24)$$

where  $k_{eq}$  is the wave number corresponding to the Hubble horizon at the equality time. The energy release per unit redshift is given by

$$\frac{dQ/dz}{\rho_\gamma} = - \int \frac{d^3 k}{(2\pi)^3} P_\gamma^i(k) \frac{d\Delta_{Q,iso}^2}{dz}. \quad (25)$$

$$\Delta_{Q,iso}(k) = \frac{\sqrt{6} c_s}{\sqrt{2}} \left( \frac{k_{eq}}{k} \right) e^{-k^2/k_D^2}. \quad (26)$$

Integration of this expression, as in Eq.(3), will give the value of  $\mu$ . In Fig.2, we show the results. For comparison, we plot the analogous relevant quantities for the adiabatic perturbations in Fig.3.

Compared to the perturbations on large scales, the perturbations on small scales do not have enough time to

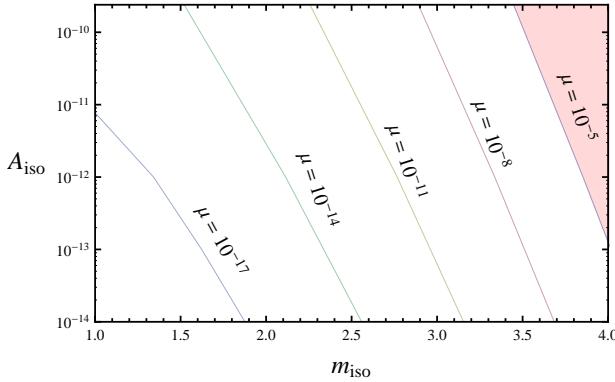


FIG. 2. Values of  $\mu$  generated by energy injection due to the damping of acoustic oscillations as a function of the isocurvature spectral index,  $m_{iso}$ , for a range of values for the amplitude of the isocurvature mode,  $A_{iso}$ . The maximum value shown for  $A_{iso}$  corresponds to the upper bound determined by WMAP of  $A_{iso} \lesssim 2.4 \times 10^{-10}$ . The shaded region has been ruled out by the COBE-FIRAS instrument.

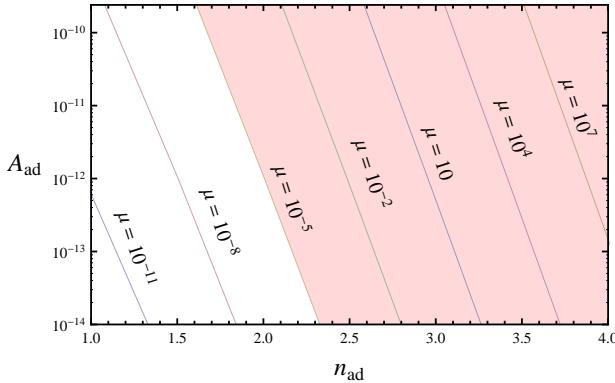


FIG. 3. Values of  $\mu$  generated by energy injection due to the damping of acoustic oscillations is shown as a function of a generic adiabatic spectral index,  $n_{ad}$ , for a range of values for the amplitude of the adiabatic mode,  $A_{ad}$ . This figure is shown for comparison with Fig. 2 in order to see the disparate contributions to  $\mu$  from adiabatic and isocurvature modes. The shaded region has been ruled out by the COBE-FIRAS instrument.

grow until they re-enter the horizon. The density perturbations on small scales are suppressed as shown Eq. (24) and the energy of the acoustic waves is also small. As a result,  $\mu$ -distortion cannot give as strong of a constraint on the isocurvature perturbations, compared to the adiabatic perturbations.

With the introduction of possible isocurvature contributions to the  $\mu$ -type distortion, one sees that a degeneracy between the various sources of the distortion may arise. This is due to the fact that  $\mu$  is simply a scalar quantity and there may be various contributions to its value. Although there may be correlations between isocurvature and adiabatic modes in various inflationary

models [18] which could possibly alleviate such a degeneracy, there may be other sources of isocurvature modes, such as cosmic defects that can contribute to the degeneracy. However, a future measurement of  $\mu$  distortion, along with complementary measurements of (or stronger bounds on) isocurvature modes from upcoming experiments such as Planck, may break the degeneracy or more strongly constrain the scenarios which can produce the distortion.

#### IV. DISCUSSION AND CONCLUSIONS

The dissipation of acoustic waves due to Silk damping creates CMB spectral distortions of the blackbody spectrum. In this paper, we have calculated  $\mu$ -type distortions from energy injection into the CMB due to the dissipation of acoustic waves from Silk damping, focusing on both the nearly-scale invariant adiabatic perturbations with a running spectral index, and isocurvature perturbations.

Fig. 1 is our main result for  $\mu$ -distortion created by the adiabatic perturbations with a running spectral index. Creation of this  $\mu$ -distortion depends on the integrated dissipation energy from the decoupling of the double Compton scatterings to the decoupling of the thermalization by Compton scatterings. We found that the dominant contribution comes from the dissipation energy at  $k \sim 100 \text{ Mpc}^{-1}$  in the case of the nearly-scale invariant adiabatic perturbations. The power spectra, which have the same amplitude at  $k \sim 100 \text{ Mpc}^{-1}$ , generate the same  $\mu$ -distortion, even though each spectral index and running index are different. As a result, a degeneracy arises between  $n_s$  and  $\alpha_s$  as shown in Fig. 1. Although the current constraint on  $\mu$ -distortion by COBE FIRAS does not give a significant constraint on both  $n_s$  and  $\alpha_s$ , the proposed PIXIE observer [12] has the potential to give a much tighter constraint, regardless of detection or non-detection of  $\mu$ -distortion. For example, the detection of  $5 \times 10^{-5} \mu$ -distortion by PIXIE will give the constraint,  $-0.004 < \alpha_s < 0$  for the scale invariant spectrum ( $n_s = 1$ ). This constraint is tighter than what is obtained by WMAP by nearly an order of magnitude. PLANCK will be able to give a constraint comparable to PIXIE, although Planck's constraint is the result of the measurement of the fluctuations on the scales larger than  $k \sim 1 \text{ Mpc}^{-1}$ . Therefore, these measurements will be complementary.

We also have evaluated  $\mu$ -distortion due to the isocurvature perturbations. The isocurvature perturbations can grow outside the horizon. Therefore, the smaller the scale of the perturbations, the lower their amplitudes. As a result,  $\mu$ -distortions created by the isocurvature perturbations are smaller compared to those generated by the adiabatic perturbations with the same amplitude and spectral index. We have found, considering the current constraint on the isocurvature amplitude from WMAP, a nearly scale-invariant isocurvature mode, which is mo-

tivated by the curvaton scenario, cannot produce significant  $\mu$ -distortion. Concerning possible cross-correlations between adiabatic and isocurvature modes then, it is conjectured that, even if a scale-invariant isocurvature mode is perfectly correlated with the adiabatic mode, the correction to the  $\mu$ -distortion compared to that arising solely from the adiabatic mode is very small. The  $\mu$ -distortion measurements at the level proposed by PIXIE would then be inefficient in placing a constraint on the correlation between the scale-invariant adiabatic and isocurvature modes. However, PIXIE can place a strong constraint on the amplitude of the isocurvature mode with a very blue spectrum ( $n < 2.5$ ), which can be generated, for example, in the axion model [16].

Finally, we comment on other potential sources of  $\mu$ -distortion. The  $\mu$ -distortion can be created by cooling of electrons, which was first computed in [13], decaying particles [19], dark matter annihilation [20], dissipation of the magnetic field energy [21], Hawking radiation of primordial black holes [22], and cosmic strings [23]. In particular,  $\mu$ -distortion due to the cooling by electrons is predicted in the standard cosmology and enters with a negative sign. Therefore, this distortion can cancel some or all of the positive  $\mu$ -distortion due to the dissipation of acoustic waves. The predicted  $\mu$ -distortion is  $-2 \times 10^{-9}$ .

In the present work we have used approximate analytic forms for the transfer functions, and therefore a more precise analysis is desirable. Although we expect the size

of the contributions from such an analysis to be negligible at the sensitivity of PIXIE, we require a more systematic analysis of  $\mu$ -distortion as PIXIE has the potential to give a precise constraint on the primordial perturbations. To address these issues, work is in progress to calculate the  $\mu$ -distortion more precisely using numerical methods.

As was point out in [14], even a non-observation at the level of  $\mu \sim \mathcal{O}(10^{-9})$  would be a sensitive cosmological probe. Furthermore, it should also be noted that if a  $\mu$ -distortion *were* observed on the order of  $\sim 10^{-7}$  or greater, the conventional cosmological picture would struggle to accommodate such a large value (see Fig.1). Thus, such an observation would lead to the tantalizing possibility of new physics<sup>1</sup>. Taken together these points underscore the exciting insights that may emerge from future CMB blackbody measurements.

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<sup>1</sup> It has also recently been shown that  $\mu$ -distortions could probe new physics via tests of Gaussianity on very small scales [24].

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